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FRACTAL MODELLING OF ECONOMIC SYSTEMS

ФРАКТАЛЬНЕ МОДЕЛЮВАННЯ ЕКОНОМІЧНИХ СИСТЕМ

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The article summarizes domestic and foreign studies of factor analysis of economic systems that seek the ideal. Their peculiarity is that they can acquire one of five states: ideal, catastrophic, current, predictive, and targeted. The tendency of dynamics of economic systems is related to the effect of existing causes and conditions of development, although after some period these causes and conditions may also change and give rise to other trends of its development. The fractal dimension of the dynamic range of the economic system is considered, which enables to predict its state in the future. Fractals simplify the complex processes occurring in the development of an economic system. It is very important for modelling economic systems allowing describing the unstable systems and processes and modelling the future of such economic systems.

Key words: *economic system, fractal analysis, dynamic range, fractal dimension.*

В статті узагальнено вітчизняні та закордонні дослідження факторного аналізу економічних систем, що прагнуть до ідеалу. Їх особливість полягає в тому, що можуть набувати один із п'яти станів: ідеальний, катастрофічний, поточний, прогнозний та цільовий. Сучасні підприємства перебувають в умовах зростаючої невизначеності і динамічності соціально економічного середовища, що характеризується складною нелінійною системою політичної, соціально-економічної, конкурентної і технологічної взаємодії. Особливої актуальності набуває проблема забезпечення управління процесами розвитку економічної системи і спрямування її до ідеалу. Зовсім новий погляд на питання управління підприємством як складною економічною системою на організаційний механізм, що покладений в основу управління, дають концепції теорії хаосу, енергетики, еволюційної теорії організації та інших системних наук. Зазначені раніше теорії досліджуються із використанням фракталів, за допомогою яких в явищах, що раніше вважалися випадковими, вдається виявити новий порядок або нову структуру. Головна причина застосування фрактального моделювання полягає у фундаментальних особливостях природи економічної системи, а саме: механізм і

структурна організація в цілому та її складових; принцип ієрархічності на всіх масштабних рівнях суспільства; систематична зміна технології виробництва; неоднорідність сировини для виробничого процесу. Управління підприємством як економічною системою є настільки складним процесом, що при його дослідженні не досить використання тільки знайомих об'єктів класичної геометрії. Настільки ж складною та іррегулярною є динаміка підприємств. Тому, при дослідженні функціонування підприємства пропонується застосовувати фрактальне моделювання. Тенденція динаміки економічних систем пов'язана з дією існуючих причин та умов розвитку, хоча після деякого періоду ці причини і умови також можуть змінитися та породити вже інші тенденції його розвитку. Розглянуто фрактальну розмірність динамічного ряду економічної системи, що дає можливість спрогнозувати її стан у майбутньому. Фрактали дозволяють спростити складні процеси, які відбуваються при розвитку економічної системи, що дуже важливо для моделювання її поведінки, дозволяють описати нестабільні системи і процеси та змоделювати майбутнє таких економічних систем.

Ключові слова: *економічна система, фрактальний аналіз, динамічний ряд, фрактальна розмірність.*

В статье обобщены отечественные и зарубежные исследования факторного анализа экономических систем стремящихся к идеалу. Их особенность заключается в том, что могут находиться в одном из пяти состояний: идеальном, катастрофическом, текущем, прогнозном и целевом. Тенденция динамики экономических систем связана с действием существующих причин и условий развития, хотя после некоторого периода эти причины и условия также могут измениться и породить уже другие тенденции его развития. Рассмотрена фрактальная размерность динамического ряда экономической системы, что дает возможность спрогнозировать ее состояние в будущем. Фракталы позволяют упростить сложные процессы, которые происходят в процессе развитии экономической системы, что очень важно для моделирования ее поведения, позволяют описать нестабильные системы и процессы, а так же смоделировать будущее таких экономических систем.

Ключевые слова: *экономическая система, фрактальный анализ, динамический ряд, фрактальная размерность.*

Problem statement. At the present stage of development, Ukraine's economy is in a crisis, caused by many reasons, including the crisis in the manufacturing sector.

Modern enterprises operate under the conditions of increasing uncertainty and dynamism of the socio-economic environment characterized by a complex non-linear system of political, socio-economic, competitive, and technological interaction. The enterprise is an economic system and, therefore, the problem of providing management of the processes of development of the economic system and directing it to the ideal becomes especially topical. At the same time, completely a new look to the issues of business management as a complex economic system, the organizational mechanism underlying the management give the concepts of theories of chaos, energy, evolutionary theory of organization and other system sciences.

Recent theories are studied using fractals to find out a new order or a new structure in phenomena that were previously considered random. Managing an enterprise as an economic system is such a complex process that it is not sufficient to use only familiar objects of classical geometry. The dynamics of enterprises are equally complex and irregular. Therefore, it is obvious that when studying the functioning of an enterprise, one should use fractal analysis, in particular, the fractal dimension of a dynamic range of factors affecting the development of the economic system.

Analysis of recent publications.

A significant contribution to the development of theoretical and methodological aspects of fractal modelling has been made by such famous national and foreign scientists as: V.Y. Ruban, I. Kornay, M.T. Matveev, M.P. Pratsovytyi, E. Feder, P. Kokh, B. Mandelbrot.

Coverage of previously unresolved parts of a general problem. Almost all economic processes are wavy, and today the problem of explaining the nature of this phenomenon exists. One of the ways to solve it can be to study the microcycles that exist in business systems of any level. For this purpose, it is enough to represent graphically the acceleration of the movement of any indicator featuring one or another aspect of the economic activity of the system in order to see the cyclicality of the process under study.

The study of the dynamics of the development of economic systems at different levels is the availability of tools that would adequately reflect the process of the development in the model. A specific feature of modelling of economic systems is the inability to conduct an experiment based on the obtained model. As a rule, the ultimate goal of modelling is to form some set of criteria used in future decision-making.

Another feature of modelling of the economic system is determined by the specificity of the source information for model construction. In most cases, this information is presented as dynamic ranges by some parameters of the studied object. The length of such ranges is usually limited to 10-12 intervals. The adequacy of the model is determined by the quality of the source information and its quantitative representation. While studying technical systems, interpolation procedures are widely used, this

in some cases compensates for the quality of the source information by increasing its quantity. For business systems, this approach cannot be applied since most of the processes occurring are strictly deterministic. The system allows fixing their parameters only at certain moments determined by the structure, and in the interval between them, there is in the state of uncertainty from the point of view of the researcher. This leads to the conclusion about the problem of improving the quality of the model due to the quantitative component of the source information obtained by interpolation. In other words, the trends of the system development should be sufficiently set in the model; attempting to compensate for the quality of the model by the amount of source information can lead to more distortion of the values of the resulting model parameters.

While dealing with the problem of studying the undulating nature of processes occurring in economic systems, most researchers use a rather limited basic set of typical models: polynomial, logarithmic, logistic, Gompertz function, and some others. These models, except for the first one, describe fairly well the processes with gradually decreasing growth rates at the end, but such processes can occur only in an idealized economic environment. The second derivative in these models is monotonically decreasing over the whole range of the research, so there is a doubt about the relevance of this model to reality. The only question that can be answered by mentioned models is the general trend of the system, but nothing more. Attempting to distinguish individual factors from the general processes described by such models is, in practice, doomed to failure.

In practical studies, the polynomial model is often used. It lacks some disadvantages of considered models, including the problem with the monotonically decreasing value of the second derivative. But it also sufficiently describes economic processes, one property of which is cyclicality. The reflection of this property is possible in a polynomial model of the degree of polynomial 9-14. Naturally, the interpretation of the obtained model parameters loses any economic content already on polynomials of 4-5 degrees. On the other hand, trying to describe complex phenomena by a quadratic or cubic polynomial, as practice shows, is not convincing enough.

There is a special position in the list of basic knowledge of the family of models based on the use of Fourier series. Paradoxically, but the use of these models to analyse economic processes is not widespread today, although there have been developed many numerical methods for approximation of Fourier coefficients. The reason for this seems to be the high degree of conservatism of economic researchers.

A brief overview of approaches to modelling economic processes based on the analysis of dynamic ranges allows drawing the following conclusion. In most studies, the modelling process comes down to finding a trend, with cyclical and random components remaining outside the model. Particular consideration should be given to the cyclic component. In the traditional interpretation, it describes long periods of relative growth and decline of the studied process. Thus, simple models can be described this way, but it has nothing to do with reality. In models that claim to describe economic processes, cyclicality should be taken into account in the trend itself and be its formal basis. In this case, cyclicality is filled with other content that is reflected in the content of the model, rather than in the form for which the source information is selected and by which some elementary model is processed.

Another problem with modelling of economic processes is that a priori they assume that the process is stationary, which is a profound mistake. The process described by the cyclic component, which in turn cyclically changes the phase and amplitude, cannot inherently be stationary. This fact is confirmed by many studies based on Wavelet analysis and Elliott waves. Unfortunately, this area is well developed enough to study other aspects of economic processes.

A common problem of these standard approaches to modelling processes occurring in the system is the implicit irrelevance of the actual process to the stationary. This can be confirmed by the fact that the model data is based on an additive component in the interpretation of the dynamic range. The approaches used in studying the dynamics of economic systems have their disadvantages and advantages.

There is an interesting problem of research of functioning of an enterprise as an economic system by means of fractals, which strives for the ideal, resulting in the possibility to find the fractal dimension of the dynamic range of the economic system. Fractals should be considered as a self-similar structure whose image is independent of scale, a recursive model, each part of which in the course of development replicates the development of the entire model as a whole. For effective functioning, an enterprise must be a system that strives for the ideal. The peculiarity of the class of systems that aspire to the ideal is that when they perform a certain task, they move to a new task that brings them even closer to the ideal. Then, an enterprise should be considered not simply as a combination of several economic objects but a system of economic objects that have a synergistic effect. Methods of fractal theory interact closely with the popular trend in the study of complex systems – synergetics. Synergetics is known to study systems that consist of a large number of components and subsystems that interact in a complex way and study their behaviour as a whole. Such systems are dynamic as they are constantly evolving and in the process of such development, they strive for the ideal.

The purpose of the article is to study the functioning of an enterprise using fractal theory. Fractal analysis in its content is a method based on the use of the principle of recursion: it proceeds from the assumption of the appropriateness of recursive communication in a dynamic series, which describes the dynamics of studied processes, allows identifying the connection between economic objects and phenomena that determine the behaviour of the studied economic system. Fractals determine the basic structure of changes in the socio-economic environment and allow us to predict with sufficient accuracy the possible tendencies of the development of economic systems that aspire to the ideal.

The main body of research. The fractal theory is very young; it appeared in the late 60s of the twentieth century. Benoit Mandelbrot, the inventor of the term «fractal», can rightfully be considered the father of fractals: «I coined the word 'fractal' using the Latin adjective 'fractus', which means irregular, recursive, fragmentary» [2, p. 53]. The first definition of a fractal was also introduced by B. Mandelbrot. Fractals are called objects that have the property of self-similarity. This means that a small fragment of the structure of such an object is similar to another, a larger fragment and even the structure as a whole. In the simplest case, a small fraction of the fractal contains information about the entire fractal. The objects of nature, such as the outlines of clouds, branches of trees, coral reefs, also have the property of self-similarity. It is not for nothing that nature «loves» fractal forms, which is emphasized precisely by the fact that they are constructed by the simple reproduction and resizing of one basic element. Fractals are used in the study of objects that have several variants of development, and the state of the system as a whole is determined by its current position. The dynamics of the enterprise's development as an economic system is closely linked to the states in which the system may reside at any time. There are five states that can be distinguished:

1. The ideal is considered as the state to which an enterprise aspires but cannot reach it, that is, its position at which it operates at no cost, which is not possible in real life.

2. We consider *a catastrophe* as a state in which an enterprise does not want to be under any circumstances.

3. *The current state* (actual achieved) is considered as the state of an enterprise in which it is currently located.

4. *The forecasted state* (expected future) is considered as the state of an enterprise in which it expects to find itself.

5. *Target* (desirable future) is considered as the state of an enterprise in which it wants to be in the future [4].

In the process of dynamics of development, an enterprise strives for the ideal. The state of the economic system is determined by the factors that influence the dynamics of this system. The study of economic systems and processes leads to the study of ranges of dynamics. The general approach to the analysis of dynamic ranges is to identify the main factors that affect the separate units of dynamic ranges. The most important components that affect the components of the dynamic ranges are considered the long-term components determining the changes in the ranges as a whole. An enterprise that is in one of five states is constantly affected by a number of factors that can be divided into three groups:

1) constant factors of non-cyclical property, such as scientific and technological progress, demographic factors, use of natural resources;

2) constant cyclical factors, both economic and social.

3) accidental and temporal factors, such as cataclysms, natural disasters [1].

The tendency of the enterprise dynamics is related to the effect of the existing causes and conditions of development, although after some period these reasons and conditions may also change and give rise to other trends of its development. Introduction to the study of random factors is quite limited, using patterns such as Gaussian or Poisson random processes.

Many experimental data have fractal statistics that can be analysed using fractal analysis methods. One of the promising areas of fractal analysis is the study of the dynamic ranges of economic systems characterized by fractal dimension. Fractal sets are determined by the Hausdorff dimension, which is larger than their topological dimension, which acquires exclusively integer values. The dimension of a single point set is considered to be zero; the dimension of the segment, straight and circle is considered equal to one; the dimension of the n-dimensional cube is n. Moreover, the topological dimension of a set is zero if for any point of a given set there is any arbitrarily small circle whose boundary does not intersect with the set. The topological dimension is n if for any point of this set there is at any arbitrary small scale the boundary of which intersects the given set by the set of dimensions *(n - 1)* and, in addition, *n* is the smallest positive number, for which this condition is fulfilled. Consider the concept of the Hausdorff dimension in the example [2].

Let the square have a broken line of unusual shape. We measure the length of a line by length segments. Then, if the broken line whose length we measure has a fractal nature, that is, a fractal line, then its total length *L* does not go to the finite boundary, as it should be for the ordinary line, but it goes to infinity by a power law. The Hausdorff dimension *D* has the following numerical value: *1 <D <2.*

In addition, for fractal dynamic series at intervals *t0 < t < T* span of the parameter *R*

$R(\tau)=\max_{1\le t\le\tau} B(t)-\min_{1\le t\le\tau} B(t)$, depends on *t* degree

$R(t)=R(t_0)\left[\frac{t}{t_0}\right]^{2-D}$, where *D*–fractal dimension of dynamic range.

Based on this expression, knowing the fractal dimension of the dynamic range and the span of the parameter at a certain interval, it is possible to predict the possible value of the span of an arbitrary parameter of the economic system in the future. The fractal dimension of *D* and its dependence on all possible parameters describing the influence of the factors determine the dynamics of the system that is seeking the ideal.

Thus, fractals can simplify the complex processes that occur in the development of the economic system, which is very important for modelling economic systems, allow us to describe unstable systems and processes, and most importantly, to model the future of such economic systems.

Conclusions. The use of fractal dimension and its dependence on the parameters that influence the factors that determine the dynamics of the studied economic system is effective in predicting the state of the economic system in the future.

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