UDC 624

MECHANICS AND MANUFACTURE OF LATTICE STRUCTURES & MATERIALS

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Many engineering structures naturally possess a periodic architecture because of the ease of their manufacture, as repetitive units can be assembled during mass production and fitted together to build a large complex structure. Such fabrication is common in traditional assemblies such as bridges, grillages, trusses, honeycombs, and so on. In these situations, periodicity is a consequence of the manufacturing process, as the production of identical units can be scaled up. Spatial periodicity is also a feature in processes that require filling a space with material. Traditional example of this could be textiles that show weave patterns. Many industrial textiles have a similar geometric feature and are technically classified as soft matter. These include woven composites, where a woven phase is usually the reinforcement phase that is embedded inside a matrix which may be a resin. Space filling in a regular pattern is also carried out during additive manufacturing, where a layer within a 2D area needs to be filled using an algorithm that drives a material dispenser. Such space filling inevitably has repetitive features in its geometry.

One of the most celebrated of power-law relationships is that concerning the apparent elasticity of honeycombs (Gibson and Ashby, 1997) for which it can be shown that in the low apparent density limit (i.e., cell walls much smaller than characteristic cell size), the apparent modulus of elasticity scales as $E{\sim}\rho^3$. The origin of this power law can be traced to the inherent mechanism of cell wall deformation, which is flexure, and the fact that bending stiffness scales according to ${\sim}t^3$, where t is the cell wall thickness, whereas the apparent density scales according to $\rho{\sim}t$. Likewise, for a triangular lattice, whose in-plane deformation is dominated by cell wall stretch, rather than flexure, the apparent modulus of elasticity obeys the power law $E{\sim}\rho$ because the stiffness in stretch scales according to $A{\sim}t$.

Here we consider a commonly encountered lattice architecture that arises naturally in additive manufacturing in addition to being the preferred geometry in biomedical scaffold applications. The elastic elements are stacked in alternating layers with orthogonal direction of their run (Figure 1). We will call such lattices as *woodpiles*, because of the association with wood logs frequently being stacked in this manner. Here we consider two variants of the stacking arrangement: those above and below a given layer are *aligned*, and those that are *staggered*. In the first case, when such elastic lattices are compressed in the stacking direction, the individual cylindrical struts are diametrically compressed. For the latter, they are primarily in bending. While the mechanics of bending for cylindrical cross-sections gives the modulus scaling as $E \sim (r/\lambda)^5$, whereas the apparent density for woodpiles is given by $\rho \sim (r/\lambda)$, which leads to the power law $E \sim \rho^5$. The details of the analysis are omitted here (see Cuan-Urquizo, et al., 2020).

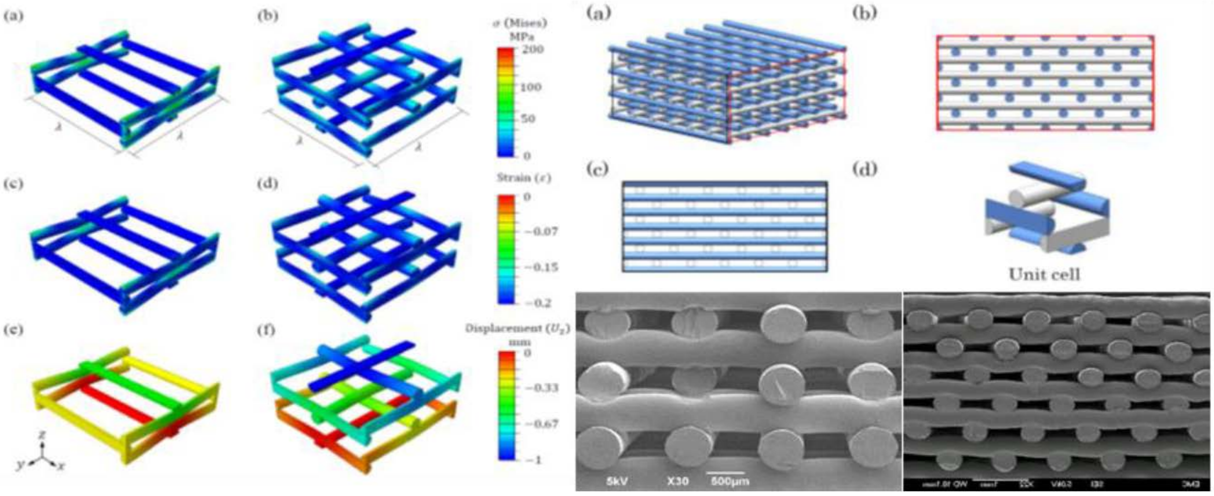

Figure 1 - Finite element computational modelling (left). Lattice geometry (top right). SEM images of aligned vs staggered arrangements in 3D printed samples of lattices.

The mechanics of the aligned arrangement is considerably more complex because the struts are neither in plane stress, nor in plane strain. A detailed analysis is mathematically cumbersome, which we avoid in favour of simple scaling argument based on dimensional consistency. The result of such a simple analysis leads to a power law for modulus-porosity relationship as $E \sim \rho^2$ (Shalchy, 2020, Shalchy and Bhaskar 2021). These power law relationships were verified for both lattice configurations using laboratory experiments on additively manufactured samples of different strut diameter. The results were consistent with the power law predictions obtained using simple ideas such as scaling and simplified mechanics. More importantly, such results are useful for industrial applications as they provide insight into the apparent elastic behaviour via simple analytical formulae.

In the above we considered a few simple cases of the static response of repetitive elastic structures under remote loading and the resulting dependence of the apparent elasticity on the porosity in the form of power laws. When elastic wave travel through such media, i.e., when such structures vibrate, the spatial periodicity leads to interesting propagation behaviour frequently encountered in solid state physics, especially in electromagnetic waves in lattices and light through crystals. While the physics of waves is considerably different in these contexts, the features of propagation that are a consequence of the spatial periodicity remain valid. Therefore, elastic waves show the so-called Band-structure with stop bands and pass bands. In a stop band, there are no propagating waves, and hence any disturbance does not propagate and is attenuated. This offers the possibility of tailoring the geometry of lattices so that vibration transmission can be suppressed. An example that we consider here is taken from engine air cooler structures (Fig 2, top left) in which fins are perforated in a lattice. One such elastic plate with periodic holes is shown in Figure 2 (top right). The lattice considered here is a square lattice.

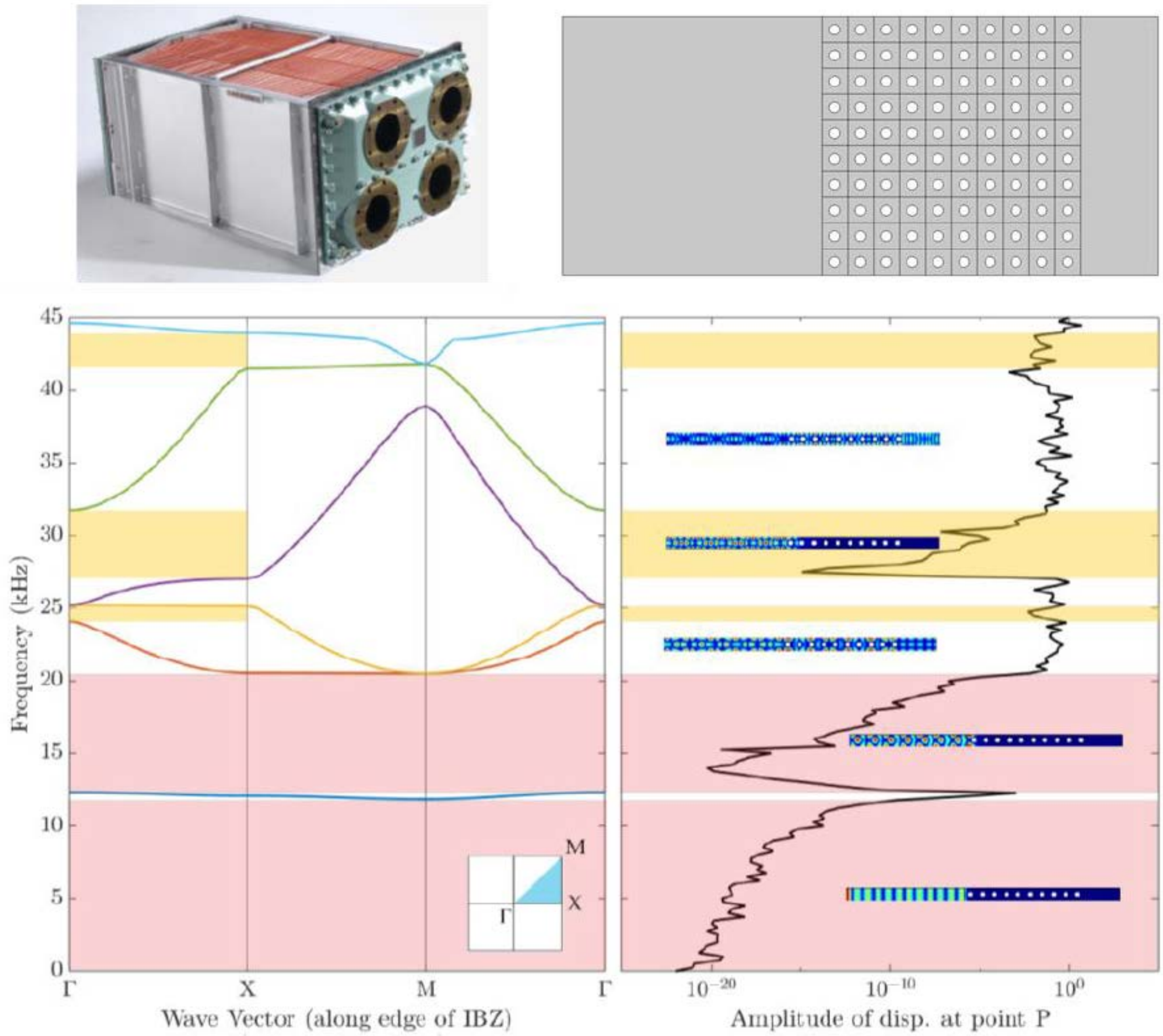

Figure 2 - Propagation of flexural elastic waves in a periodically perforated plate. The computations were carried out in the Comsol environment

The horizontally running bands denote stop bands (see, e.g., Kittel, 2004), where there is wave attenuation. Figure 2 shows the propagation behaviour for flexural waves in a plate with lattice perforations. It is customary to show the dispersion of such waves along the edges of the so-called irreducible Brillouin zones (IBZ) in the wave-number space that contain information about all possible propagation (Brillouin, 1946). The attenuation is confirmed on the right side of this figure. Note that there are partial stop bands in certain directions (along Γ-X edge) where the attenuation efficiency is only moderate. In a thin pass band, there is considerable amplification, by contrast.

Summary. A host of industrial parts and additively manufactured materials possess the feature of spatial periodicity. This inherent periodicity can be exploited to simplify analysis. In particular, the apparent properties can be derived by considering a unit cell subjected to remote loading. Results in the form of power laws offer practical usefulness to practicing engineers. The possibility of producing a designer material can also be explored. Unusual dynamic phenomena in structural vibration can also be studied, which, in turn, can be used for vibration attenuation. Details of the work cited here will be presented elsewhere.

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