TECHNICAL SCIENCES

UDC 685.31.02

CONSTRUCTION OF LATTICE PACKINGS OF TWO FLAT GEOMETRIC OBJECTS WITH DIFFERENT CONFIGURATION OF THE EXTERNAL CONTOUR

Chuprynka Viktor Ivanovich
Doctor of Technical Sciences, Professor
Chuprynka Natalia Viktorovna
Candidate of Technical Sciences, Associate Professor
Naumenko Bohdan Vitaliyovych
Vasylenko Oleksiy Leonidovych
graduate students
Kyiv National University of Technology and Design,
Kyiv, Ukraine
Syrotynskyi Oleksandr Artemovych
Candidate of Technical Sciences, Associate Professor
National University of Water and Environmental Engineering,
Rivne, Ukraine

Abstract

The method of construction of the latticed piling for two plane geometric objects with different configuration of outer contours and search is in-process examined the densest from the possible latticed piling for these objects. To do this, the mathematical formulation of the problem, highlighted the structural components of the problem and given their analytical description. Analytical expression is got for a goal function and the method of her optimization is offered.

The proposed method of construction of lattice packing's has been implemented in a software product for the automatic construction of lattice packing's for two planar geometries with a different configuration of the outer contour and the search of the densest lattice packing's permissible for these objects.

Keywords: styling, hodograph, dense combination, the objective function, the optimization, software.

Introduction. Rational and economic spending of material and energy resources, as well as protection of the environment from pollution have always been and are priority directions in the development of Ukraine. And for this, it is necessary to reduce the amount of waste. Thus, materials make up more than 80% of the cost of shoes, and the technological features of shoe production lead to the fact that only the waste of cutting shoe materials makes up more than 20%, so the importance of rational use of materials is obvious.

Aim. Development of algorithms and a software product for the design of dense lattice stacks for two types.

Materials and methods. The object of research is the technological process of cutting rectangular materials into haberdashery details. The subject of the study is the automated design of rational schemes for cutting rectangular materials into haberdashery details. The research methods are based on the basic provisions of the haberdashery production technology, mathematical modeling, the theory of lattice laying and the methods of computer graphics, computational mathematics and analytical geometry.

Results and discussion. Consider objects S1 and S2 on the plane. Let $int\ S = S - S^\wedge$, where $S^\wedge$ is the boundary of the object S.

The objects S_1 and S_2 do not intersect if

$$int\ S_1 \cap int\ S_2 = 0 \quad (1).$$

If at the same time the condition

$$S_1 \cap S_2 \neq 0, \quad (2)$$

then the objects S_1 and S_2 are said to be densely placed. Densely placed objects do not have common interior points, but they do have common boundary points.

The system of objects S_i , $i=1..p$, form a laying on the plane if for each pair of objects from this system the conditions of their mutual non-intersection are satisfied:

$$int\ S_n \cap int\ S_m = 0,\ n\ m,\ n,m=1..p \quad (3)$$

and for any object S_i , i=1..p, there is at least one object S_q, where q ϵ[*1..p*], *p≠i*, which touches the object S_i.

Consider objects S_1 and S_2 on the plane. Set of view vectors:

$\boldsymbol{r_1}=n\boldsymbol{a_1}+m\boldsymbol{a_2}$ and $\boldsymbol{r_2}=n\boldsymbol{a_1}+m\boldsymbol{a_2}+\boldsymbol{g}$, where $n,m=0, \pm1, \pm2, \pm3,\ldots\pm k\ldots$, $\boldsymbol{a_1,a_2}$, are linearly independent vectors, we will call a double lattice with a basis $\boldsymbol{a_1,a_2}$,and a lattice shift vector $\boldsymbol{g}$ and denote by $\boldsymbol{W}=\boldsymbol{W}(\boldsymbol{a_1,a_2,g})$. The absolute value of the determinant, which is composed of the base vectors of the double lattice, will be called the determinant of the double lattice and denoted by ***det W.***

Consider a system of objects $\bigcup\limits_{n.m} S_1^{nm}$ and $\bigcup\limits_{n.m} S_2^{nm}$, where $n,m=0, \pm1, \pm2, \pm3,\ldots\pm k\ldots$, which consist of $S_1^{nm}=S_1+n\boldsymbol{a_1}+m\boldsymbol{a_2}$ и $S_2^{nm}=S_2+n\boldsymbol{a_1}+m\boldsymbol{a_2}+\boldsymbol{g}$, $n,m=0, \pm1, \pm2, \pm3,\ldots\pm k\ldots$, objects S_1 and S_2 to the vectors of the double lattice $\boldsymbol{W}=\boldsymbol{W}(\boldsymbol{a_1,a_2,g})$[1]. If this system is a stacking, then such a stacking is called a stacking of objects S_1 and S_2, performed on a double lattice $\boldsymbol{W}=\boldsymbol{W}(\boldsymbol{a_1,a_2,g})$. The double lattice $\boldsymbol{W}=\boldsymbol{W}(\boldsymbol{a_1,a_2,g})$ in this case is admissible for stacking objects S_1 and S_2. A fragment of double lattice laying is shown in fig. 1.

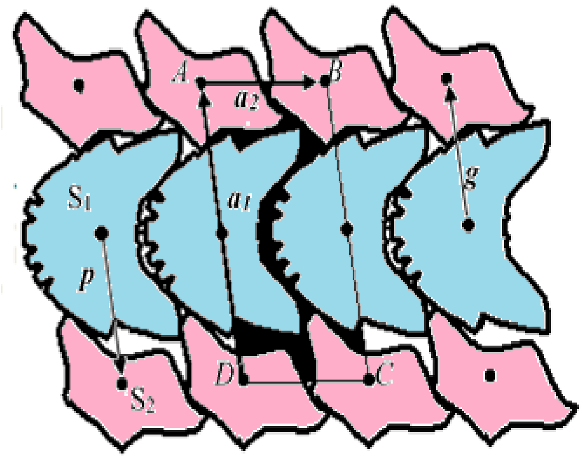

Fig. 1. Fragment of a double lattice laying for flat geometric objects of the form S_1 and S_2

A double lattice is two identical single $\Lambda_1=\Lambda(\boldsymbol{a_1,a_2})$ and $\Lambda_2=\Lambda(\boldsymbol{a_1}+\boldsymbol{g}$, $\boldsymbol{a_2}+\boldsymbol{g})$, which are shifted relative to each other by the lattice displacement vector $\boldsymbol{g}$. At the nodes of the lattice Λ_1, objects S_1 are placed, and at the nodes of the lattice Λ_2, objects S_2 are placed. The absolute value of the determinant composed of lattice vectors will be called the determinant of the lattice $\boldsymbol{W}$ and denoted ***det W***, where :

$$\det \boldsymbol{W} = |[\, \boldsymbol{a_1} \times \boldsymbol{a_2}\,]| = \left\| \begin{bmatrix} a_{1x} & a_{1y} \\ a_{2x} & a_{2y} \end{bmatrix} \right\| = \left| a_{1x} a_{2y} - a_{2x} a_{1y} \right|. \quad (4)$$

Density $\delta_s(\boldsymbol{W})$ of lattice laying can be characterized using the relation[2]:

$$\delta_s(\boldsymbol{W}) = (|S_1| + |S_2|) / \det \boldsymbol{W}, \quad (5)$$

where $|S_1|$ and $|S_2|$ are, respectively, the areas of flat geometric objects S_1 and S_2, ***det W*** is the determinant of the lattice $\boldsymbol{W}=\boldsymbol{W}(\boldsymbol{a_1}, \boldsymbol{a_2}, \boldsymbol{g})$, along which the laying is performed. From the above relation, it can be seen that the density $\delta_s(\boldsymbol{W})$ of the lattice laying is the higher, the smaller the area of the parallelogram, the sides of which are the base vectors of the lattice $\boldsymbol{a_1}$ and $\boldsymbol{a_2}$.

Mathematical statement of the problem „Laying”. Among the set of admissible double lattices $\boldsymbol{W_i}=\boldsymbol{W}(\boldsymbol{a^i_1}, \boldsymbol{a^i_2}, \boldsymbol{g^i})$ $i=1,2..q$,, dense packings of flat geometric objects S_1 and S_2 (object S_1 rotated by an angle α ($0 \le \alpha \le 180^0$), object S_2 by angle β ($0 \le \beta \le 180^0$ relative to their initial position) , find such a lattice $\boldsymbol{W^*}=\boldsymbol{W}(\boldsymbol{a^*_1}, \boldsymbol{a^*_2}, \boldsymbol{g^*})$, for which

$$\delta_s(\boldsymbol{W^*}) = (|S_1| + |S_2|) / \det \boldsymbol{W^*} = max(\delta_s(\boldsymbol{W^i})), \text{ or } \det \boldsymbol{W^*} = min(\det \boldsymbol{W^i})$$

where $|S_1|$ and $|S_2$ are, respectively, the areas of flat geometric objects S_1 and S_2.

Mathematical model of the task of designing the densest lattice stacking. The mathematical model of the problem posed should reflect the geometric shape of flat geometric objects, the system for placing flat geometric objects on the plane, the conditions for mutual non-intersection of flat geometric objects in the stacking. To formalize this problem and develop its mathematical model, it is necessary to make its decomposition.

In the problem of constructing the densest lattice packing, the following structural components can be distinguished:

- analytical presentation of information about external contours, placed flat geometric objects;
- parameters that determine the position of a flat geometric object on a plane;
- analytical description of the conditions of mutual non-intersection of flat

geometric objects in the stack;

- analytical description of the system for combining flat geometric objects in the stack;

- mathematical description of the set of feasible solutions to the problem;

- analytical representation of the goal function. Below we will dwell on each of the components of the tasks, given that they must ensure the adequacy, universality and economy of the mathematical model.

Analytical description of the outer contour of a flat geometric object. To unambiguously display the position of a flat geometric object S in a stack and generate a set of allowable dense stacks, it is necessary to analytically describe the outer contour of a flat geometric object and determine the parameters that uniquely reflected its position on the plane.

The contours of flat geometric objects can have a complex outer contour and it is impossible to describe them analytically in the form of a mathematical function $F(x, y) = 0$ in most cases. Therefore, we will approximate flat geometric objects S in the form of polygons Sm with a given accuracy ε. To uniquely determine the outer contour of the polygon Sm, it is sufficient to know the coordinates of the vertices A_i (X_i, Y_i), where $i = 1,2 \dots n$ and $X_1 = X_n$, $Y_1 = Y_n$ (fig. 2).

Parameters that determine the position of a flat geometric object on a plane. To unambiguously display the position of a flat geometric object *S* on the plane, it is necessary to know the coordinates of the pole, any fixed point inside the flat geometric object (Xp_k, Yp_k) in the *XOY* coordinate system associated with the plane, and the angle of rotation θk of the flat geometric object relative to its initial position.

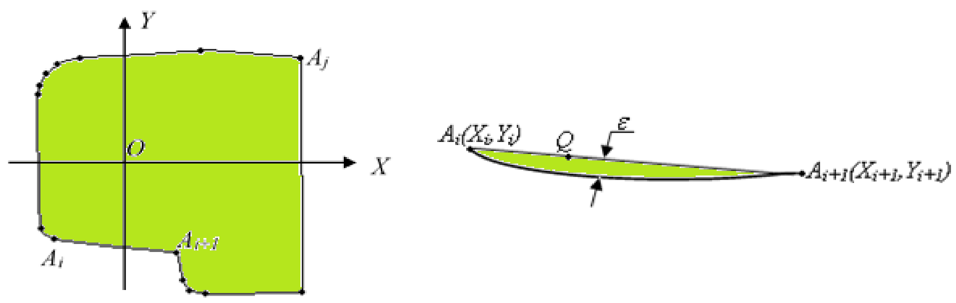

Fig. 2. Approximation of the outer contour of a flat geometric object

Analytical description of the conditions of mutual non-intersection of flat geometric objects in stacking. Using the hodograph apparatus of the dense placement vector function (DPVF) [3-4], we have the ability to control the relative position of flat geometric objects in the stacking.

Mathematical description of the set of feasible solutions for the problem of designing the densest lattice stacking. It is obvious that the set of admissible solutions of this problem will be the set of admissible double lattices $\boldsymbol{W^i}=\boldsymbol{W}(\boldsymbol{a^i_1}, \boldsymbol{a^i_2}, \boldsymbol{g^i})$, where $i=1,2..q$.

Having determined the range of parameters $\boldsymbol{a^i_1}, \boldsymbol{a^i_2}, \boldsymbol{g^i}$ of double lattices $\boldsymbol{W^i}$, we will uniquely determine the set of admissible solutions. Double lattice laying is uniquely determined by the mutual arrangement of three adjacent rows (Fig. 3).

The search for a set of feasible solutions for the problem of constructing the densest lattice stacking of flat geometric objects S_1 and S_2 (flat geometric objects S_1 are rotated by an angle α, and flat geometric objects S_2 are rotated by an angle β relative to their initial position) (Fig. 3) is considered in detail in the work [5].

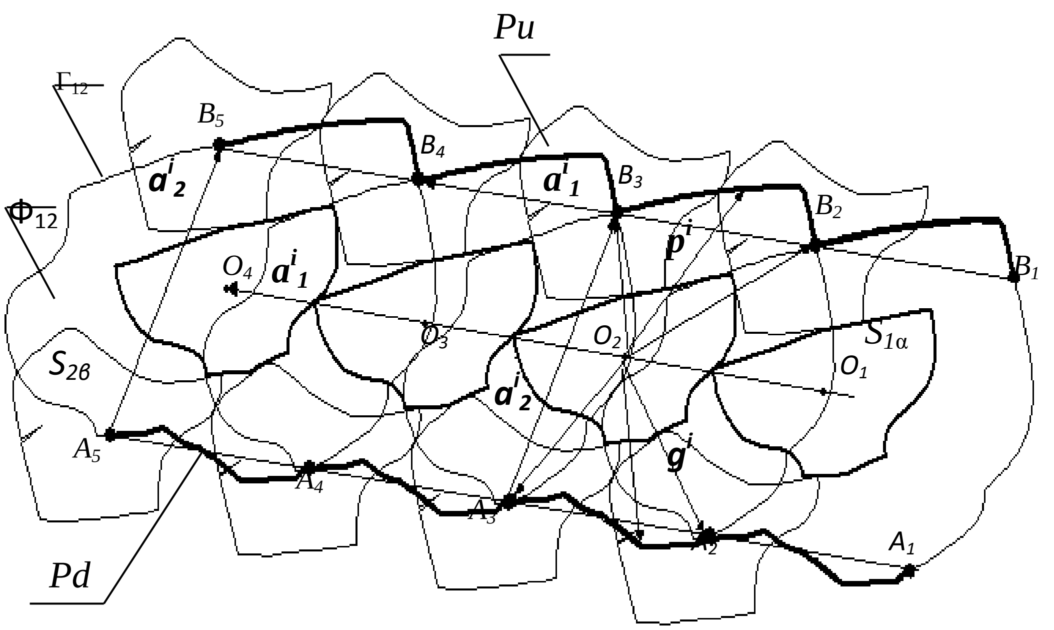

Fig. 3. Value ranges of the vectors pi and gi for the problem "Laying"

Analytical representation of the goal function for the problem of designing the densest lattice packing for flat geometric objects of the form S_1 and S_2. Since the mathematical model of the problem of designing the densest lattice packing for flat geometric objects of the form S_1 and S_2 is a double lattice, the density characterize using relation (5). The areas | S_1| and | S_2| of the geometric objects S_1 and S_2 are constant in this ratio, therefore the density δ_s*(W)* of the lattice laying will be determined by the lattice determinant, the value of which is equal to the area of the parallelogram, the sides of which are the base vectors of the lattice ***a₁*** and ***a₂***. Then from expression (5) it is obvious that the density δ_s*(W)* of the lattice laying will be the higher, the smaller the area of this parallelogram. That is, the determinant ***det W*** of the double lattice ***W*** will be the objective function.

Since we have the analytical form of the DPVF $\boldsymbol{\Gamma_{11}= r_{11}(\theta)}$ and $\boldsymbol{\Gamma_{12}= r_{12}(\theta)}$, and the vectors $\boldsymbol{a_1= f_1(r_{11}(\theta))}$ and $\boldsymbol{a_2=f_2(r_{12}}(\theta))$, we will find analytical expressions for the lattice vectors $\mathbf{a_1}$ and $\mathbf{a_2}$. Let the DPVF $\boldsymbol{\Gamma_{11}}$ and $\boldsymbol{\Gamma_{12}}$ have the following analytical form:

$$\Gamma_{11}: \begin{cases} xg^{11} = (Xg_{i+1}^{11} - Xg_i^{11})t_i - Xg_i^{11} \\ yg^{11} = (Yg_{i+1}^{11} - Yg_i^{11})t_i - Yg_i^{11} \end{cases}, \text{ where } i = 1,2...n_g^{11} \text{ and } t_i \in [0,1], \quad (6)$$

$$\Gamma_{12}: \begin{cases} xg^{12} = (Xg_{i+1}^{12} - Xg_i^{12})\tau_i - Xg_i^{12} \\ yg^{12} = (Yg_{i+1}^{12} - Yg_i^{12})\tau_i - Yg_i^{12} \end{cases}, \text{ where } i = 1,2...n_g^{12} \text{ and } \tau_i \in [0,1]. \quad (7)$$

Then the vectors ***a₁, p*** , ***g*** can be represented as follows:

$$\boldsymbol{a_1}: \begin{cases} x_{a1} = (Xg_{i+1}^{11} - Xg_i^{11})t_i - Xg_i^{11} \\ y_{a1} = (Yg_{i+1}^{11} - Yg_i^{11})t_i - Yg_i^{11} \end{cases}, \text{ where } i = 1,2...n_g^{11} \text{ та } t_i \in [0,1], \quad (8)$$

$$\boldsymbol{p}: \begin{cases} x_p = (Xg_{j+1}^{12} - Xg_j^{12})\tau_j - Xg_j^{12} \\ y_p = (Yg_{j+1}^{12} - Yg_i^{12})\tau_j - Yg_j^{12} \end{cases}, \text{ where } j = 1,2...n_g^{12} \text{ and } \tau_j \in [0,1], \quad (9)$$

$$\boldsymbol{g}: \begin{cases} x_g = (Xg_{k+1}^{12} - Xg_k^{12})\tau_k - Xg_k^{12} \\ y_g = (Yg_{k+1}^{12} - Yg_k^{12})\tau_k - Yg_k^{12} \end{cases}, \text{ where } k = 1,2...n_g^{12} \text{ and } \tau_k \in [0,1]. \quad (10)$$

Hence, the goal function has the following form:

$$\det \boldsymbol{W} = |[\, \boldsymbol{a_1} \times \boldsymbol{a_2} \,]| = \left\| \begin{bmatrix} a_{1x} & a_{1y} \\ a_{2x} & a_{2y} \end{bmatrix} \right\| = \left\| \begin{bmatrix} a_{1x} & a_{1y} \\ x_p - x_g & y_p - y_g \end{bmatrix} \right\| =$$

$$= \left| a_{1x}(y_p - y_g) - (x_p - x_g)a_{1y} \right| = \left| a_{1x}y_p - a_{1x}y_g - x_p a_{1y} + x_g a_{1y} \right| = F(t_i, \tau_j, \tau_k, i, j, k) =$$

$$= \left| A_{ij}t_i\tau_j + B_{ik}t_i\tau_k + (C_{ij} + D_{ik})t_i + E_{ij}\tau_j + F_{ik}\tau_k + L_{ijk} \right| , \quad (11)$$

$$
\begin{aligned}
A_{ij} &= (Xg_{i+1}^{11} - Xg_i^{11})(Yg_{j+1}^{12} - Yg_j^{12}) - (Yg_{i+1}^{11} - Yg_i^{11})(Xg_{j+1}^{12} - Xg_j^{12}),\\
B_{ik} &= (Yg_{i+1}^{11} - Yg_i^{11})(Xg_{k+1}^{12} - Xg_k^{12}) - (Xg_{i+1}^{11} - Xg_i^{11})(Yg_{k+1}^{12} - Yg_k^{12}),\\
C_{ij} &= Xg_j^{12}(Yg_{i+1}^{11} - Yg_i^{11}) - Yg_j^{12}(Xg_{i+1}^{11} - Xg_i^{11}),\\
\text{where}\qquad D_{ik} &= Yg_k^{12}(Xg_{i+1}^{11} - Xg_i^{11}) - Xg_k^{12}(Yg_{i+1}^{11} - Yg_i^{11}),\qquad (12)\\
E_{ij} &= Yg_i^{11}(Xg_{j+1}^{12} - Xg_j^{12}) - Xg_i^{11}(Yg_{j+1}^{12} - Yg_j^{12}),\\
F_{ij} &= Xg_i^{11}(Yg_{k+1}^{12} - Yg_k^{12}) - Yg_i^{11}(Xg_{k+1}^{12} - Xg_k^{12}),\\
L_{ijk} &= Xg_i^{11}Yg_j^{12} + Yg_i^{11}Xg_k^{12} - Xg_i^{11}Yg_k^{12} - Yg_i^{11}Xg_j^{12}.
\end{aligned}
$$

As can be seen from equations (8-12), the goal function ***det W*** is a linear function of three $t_i \in [0,1]$, $\tau_j \in [0,1]$, $\tau_k \in [0,1]$ and three discrete parameters *i, j, k* ($i \in 1,2...n_g^{11}$, $j \in 1,2...n_g^{12}$, $k \in 1,2...n_g^{12}$) has the form:

$$\det \boldsymbol{W} = F(t_i, \tau_j, \tau_k, i, j, k) = \left| A_{ij} t_i \tau_j + B_{ik} t_i \tau_k + (C_{ij} + D_{ik}) t_i + E_{ij} \tau_j + F_{ik} \tau_k + L_{ijk} \right|. \quad (13)$$

Then the local extremum of this function can only be at the limiting values of the variables, that is, for

$$t_i = \begin{cases} 0 \\ 1 \end{cases};\ \tau_j = \begin{cases} 0 \\ 1 \end{cases};\ \tau_k = \begin{cases} 0 \\ 1 \end{cases}. \quad (14)$$

Thus, we do not need to sort through all the values for the variables $t_i \in [0,1]$, $\tau_j \in [0,1]$, $\tau_k \in [0,1]$, but only those values where the local extremum for the goal function (13) is reached, among which we choose the minimum value as a solution to the optimization problem of constructing dense packings. The conducted studies of the goal function made it possible to present a theoretically substantiated method for finding the extremum of the goal function in the range of acceptable values.

By minimizing the ***det W*** function of the target (parallelogram area ***ABCD*** , we increase stacking density and minimize waste(Fig. 1).

The proposed method for constructing lattice stacks was implemented in a software product for automatically searching for the densest possible lattice stack.

The software product allows you to calculate the parameters of the most efficient stacking and build these lattice stacking, determine the most efficient stacking. It is implemented in the Delphi programming environment for the Windows operating system. On fig.4 shows an example of the obtained dense lattice stacking for two flat geometric objects of various configurations using the developed software product.

Dense stacking parameters:

Sdet=5.35 sq. dec.

Spar=6.36 sq. dec.

Dense stacking=84.18%

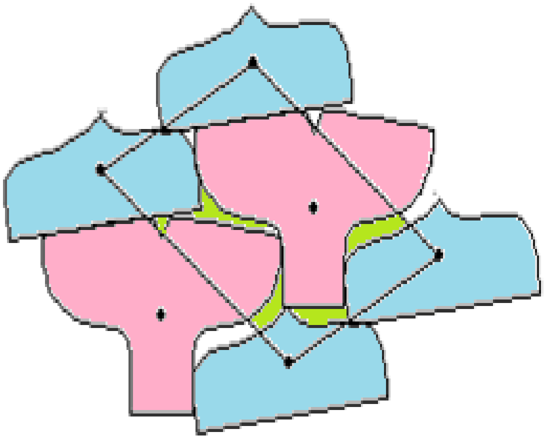

Fig. 4. An example of a dense lattice stacking for two flat geometric objects of various configurations

Conclusions. The proposed method for constructing lattice packings for two flat geometric objects of various configurations is implemented in the Delphi programming environment for the Windows operating system. The developed software product has a friendly interface and does not require special knowledge of computer science when working with it.

REFERENCES

1. Stoyan YU.G. Metody i algoritmy razmeshcheniya ploskikh geometricheskikh ob"yektov/ YU.G. Stoyan, N.I. Gil'.– Kiyev: Naukova dumka, – 1976,– 242 s.
2. Fesenko A.G. Metody i algoritmy naiplotneyshey reshetchatoy ukladki ploskikh ob"yektov: avtoref. dis. na soisk. nauch. stepeni kandidata fiziko-matematicheskikh nauk: spetsial'nost' 01.008. „Vychislitel'naya matematika” / A.G. Fesenko. – Kiyev, 1981, – 24 s
3. Utina L.S. Postroyeniye godografa funktsii plotnogo razmeshcheniya dvukh vypuklo-vvognutykh mnogougol'nikov / L.S. Utina. // Izvestiya vysshikh uchebnykh zavedeniy. Tekhnolog. legkoy promyshlennosti. – 1979. – №1. – S. 51-55. – №3. – S. 62-66

4. Utina L.S. Postroyeniye godografa funktsii plotnogo razmeshcheniya odnosvyaznykh figur / L.S. Utina, V.A. Skaternoy // RFAP, Institut kibernetiki AN USSR, registratsionnyy № 5139, – Kiyev, 1979,– 36 s.

5. Chuprynka V. I. Algorithm for finding rotation angles α and β, respectively, for stationary and moving flat geometric objects when constructing dens stacks of these objects / V. I Chuprynka., N. V Chuprynka., B. V. Naumenko, O. L.Vasylenko O. L., O. A. // Innovations and prospects in modern science. Proceedings of the 7th International scientific and practical conference. SSPG Publish. Stockholm, Sweden. 2023. Pp. 40-46. URL: https://sci-conf.com.ua/vii-mizhnarodna-naukovo-praktichna-konferentsiya-innovations-and-prospects-in-modern-science-3-5-07-2023-stokgolm-shvetsiya-arhiv/.