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ON THE EXPLICIT FORM OF SOME LIMIT FUNCTIONALS OF GENERALIZED RANDOM PROCESSES WITH INDEPENDENT VALUES

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The case of pairwise singularity of probability measures in the proper probability space makes it possible to unmistakably distinguish hypotheses about a specific type of random process to which these measures correspond [1]. In particular, it applies to the processes of economics, medicine, and production processes, for which a significant change in the numerical characteristics of the observed processes may indicate significant changes in the situation in the field under study. In some cases, the above conditions can be obtained using certain limit theorems. The latter are called Levy □ Baxter type theorems. An extensive literature is devoted to such theorems for ordinary processes. The case of generalized processes has been studied much less in this regard. In this report, we give the formulation of a limit theorem of the specified type. The condition of a singularity of probability measures, which is obtained with the help of the given result, was formulated in paper [1] (see also [2,3]). If these conditions are met, it is possible to determine which process is being observed. Let $C_0^\infty(T)$be the space of real finite infinitely differentiable functions on the set *T*, ⌊ = ⌊(*Π*), $\in C_0^\infty([0,1])$ be a real normal (Gaussian) generalized random process with independent values. Independence of process values means that the random variables ⌊(*Π*), ⌊() are independent, if the supports of *Π*, do not have a common interior points. Without limiting the conditions in principle, it can be assumed [1] that the covariance functional $B(\varphi,)$ of the specified (generalized) process is presented in the form

$$B(\varphi,) = \sum_{k=0}^{N} \int_0^1 R_k(x)\varphi^{(k)}(x)\quad {}^{(k)}(x)dx, \quad \varphi, \in C_0^\infty([0,1]), \quad (1)$$

where $R_k(x)$ are continuous functions on the segment [0,1], *i* =1,2.

Further, when constructing a family of basic functions for expression (1), it will be more convenient for us to proceed from a two-parameter test function family of the form

$$A = \{\chi_{t,h} = \chi_{t,h}(\cdot) | t \in R, h \in (0,1), \chi_{t,h} \subset [t, t+h]\}, \qquad (2)$$

assuming the formation of Baxter sums by the equality

$$\square_{k,n} = \chi_{t,h}\Big|_{t=\frac{k}{b(n)},h=\frac{1}{b(n)}}, k = 0,1,\ldots, b(n)-1, n \geq 1, \qquad (3)$$

where *b(n)* is an integer function, $b(n)\, n \to \infty \to \infty$.

A family of test functions (2) is called O_2 type family if for these functions

$$\int_t^{t+h} \chi_{t,h}^2(x)dx = h + o(h), h \to 0+ \text{ uniformly over } t \in \square.$$

A family of test functions (3) is called o_2 type family if for these functions

$$\int_t^{t+h} \chi_{t,h}^2(x)dx = o(h), h \to 0+ \text{ uniformly over } t \in \square.$$

Let $N \geq 0$ be an integer. A family of test functions $\{\square_{t,h} | t \in R, h \in (0,1), \alpha_{t,h} \subset [t, t+h]\}$ is called $O_2^{(N)}$ type family if:

1) The family $\{\square_{t,h}^{(N)}\}$ has type O_2. 2) Each family of derivatives $\{\square_{t,h}^{(l)}\}$, where $l < N$, is of type o_2.

Theorem Let ⌊ be a generalized Gaussian random process having zero mathematical expectation and a covariance functional whose restriction to [0,1] has the form (1), where functions $R_k, 0 \leq k \leq N$ are continuous on an interval $[0,1]$. Then for any family of test functions $\{\square_{t,h} = \chi_{t,h}(\cdot) | t \in R, h \in (0,1), supp\chi_{t,h} \subset [t, t+h]\}$ having $O_2^{(N)}$ type

$$S_n(\xi) = \sum_{k=0}^{b(n)-1} (\xi, \chi_{k,n})^2 \to \int_0^1 R_N(x)dx \qquad (4)$$

in the square mean as $n \to \infty$. If the series $\sum_{n=1}^{\infty} \frac{1}{b(n)}$ converges, then (4) converges almost surely.

References

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