УДК 621.372.41: 621.3.011.71: 621.3.018

# RESONANT PHENOMENA IN BRIDGE CIRCUITS: DUALITY, AMPLITUDE, AND PHASE RESONANCE

**Blokhin D.** - group BZKI-23, Bachelor, *danialsmitua@gmail.com*
**Panasiuk I.** - D.Sc. (Eng.), Professor, *fiit@knutd.edu.ua*
**Demishonkova S.** - Ph.D. (Eng.), Associate Professor, *fiit@knutd.edu.ua*
*Kyiv National University of Technologies and Design*

***Abstract.*** This paper presents a theoretical investigation of the frequency characteristics of alternating current (AC) bridge circuits, considered as two-terminal networks. The study focuses on the manifestation of resonant phenomena - particularly amplitude and phase resonance — and the role of structural dualities inherent in bridge configurations. Despite extensive prior research on bridge applications, the frequency behaviour of such circuits remains insufficiently characterized. The proposed analysis provides a unified mathematical framework for describing impedance and admittance properties of bridge circuits and establishes the conditions for the occurrence of resonance. The obtained results extend the potential applications of bridge circuits to analog signal processing, specifically in the design of frequency-selective filters.

***1. Introduction***. Bridge circuits, commonly known as electrical bridges, are four-arm configurations interconnected in the form of a square or diamond. They have been widely employed in electrical engineering for precision measurement and signal analysis. The fundamental concept involves connecting a power source (either direct or alternating current) to one diagonal of the bridge and a measuring device – such as a galvanometer, voltmeter, or oscilloscope - to the other. Well-known examples include the Wheatstone, Kelvin, Schering, Maxwell, Hay, and Wien bridges, among others [1].

In alternating-current bridges that incorporate reactive elements (capacitors and inductors) in their arms, resonance phenomena arise. These resonances manifest as current or voltage resonance, depending on the configuration. However, the frequency-dependent behaviour of such bridges has not been systematically described. The objective of this work is to fill this gap by deriving analytical expressions for the amplitude–

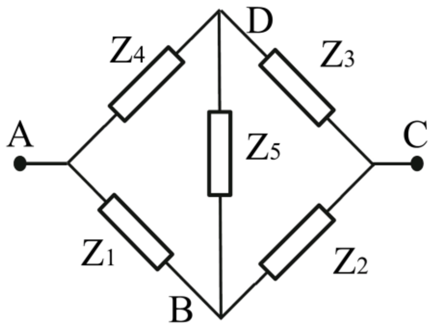


Fig.1. A two-terminal network in the form of a bridge

frequency and phase–frequency characteristics of AC bridge networks and by revealing their underlying duality relations [6,7].

***2. Theoretical Background***. The bridge under study can be represented as a two-terminal network with impedances $Z_1$–$Z_5$ (Fig. 1) forming the arms and diagonal. An AC voltage of frequency ω is applied between terminals A and C, and the current depends on the composite impedance of the network.

Using Δ–Y and Y–Δ transformations, analytical expressions for the total impedance $Z_{AC}$ and admittance $Y_{AC}$ can be obtained [1]:

$$Z_{AC} = \frac{Z_1Z_2(Z_3+Z_4)+Z_3Z_4(Z_1+Z_2)+Z_5(Z_1+Z_2)(Z_3+Z_4)}{(Z_1+Z_4)(Z_2+Z_3)+Z_5(Z_1+Z_2+Z_3+Z_4)} \tag{1}$$

$$Y_{AC} = \frac{Y_1Y_2(Y_3+Y_4)+Y_3Y_4(Y_1+Y_2)+Y_5(Y_1+Y_4)(Y_2+Y_3)}{(Y_1+Y_2)(Y_3+Y_4)+Y_5(Y_1+Y_2+Y_3+Y_4)}. \tag{2}$$

The obtained relations exhibit a high degree of symmetry due to the structural properties of the bridge. Two geometric symmetries – reflections about diagonals BD and AC – correspond to equivalent dual transformations. A third form of duality arises from the classical relationships between current and voltage, impedance and admittance, and inductance and capacitance. Together, these three dualities constitute an Abelian group of eight elements, enabling simultaneous analysis of multiple equivalent bridge configurations through a single analytical function.

***3. Bridge with Reactive Components***. Consider a bridge containing inductance L and capacitance C in opposite arms (Fig. 2) [6,7]. The impedances of the arms and diagonal can be expressed as functions of these elements. $Z_1 = \frac{1}{j\omega C}$ , $Z_2 = R_1$ $Z_3 = j\omega L$ , $Z_4 = R_2$, $Z_5 = R_3$. By introducing dimensionless parameters a, b, and c (representing quality factors) and the reduced frequency $x = \omega/\omega_0$: $x = \omega\sqrt{CL}$ , $a = R_1\sqrt{C/L}$ , $b = R_2\sqrt{C/L}$ , $c = R_3\sqrt{C/L}$ the system can be described by a normalized impedance function [2,3,4]:

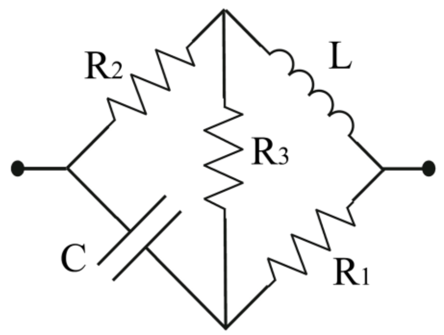

Fig.2 Oscillator bridge

$$Z_{AB} = \sqrt{\frac{L}{C}} \cdot f(x,a,b,c) \tag{3}$$

$$f(x,a,b,c) = \frac{(a+b+c+abc)+j\big(a(b+c)x-b(a+c)/x\big)}{(1+ab+ac+bc)+j\big((b+c)x-(a+c)/x\big)} \tag{4}$$

The frequency-independent constant in the impedance expression is referred to as the characteristic impedance. The frequency-dependent behaviour is governed by a dimensionless function *f(x, a, b, c),* whose properties reflect the bridge symmetries. Among the eight dualities identified earlier, four remain when specific L and C elements are introduced, allowing reduced analysis without loss of generality.

I. $a \Leftrightarrow b$ , $c \Leftrightarrow c$ , $x \Leftrightarrow -1/x$ , $f(x,a,b,c) = f(-1/x,b,a,c)$

II. $a \Leftrightarrow 1/a$ , $b \Leftrightarrow 1/b$ , $c \Leftrightarrow 1/c$ , $x \Leftrightarrow x$ , $f(x,1/a,1/b,1/c) = 1/f(x,a,b,c)$

***4. Phase Resonance***. Phase resonance occurs when the phase shift between the input voltage and the current passing through the bridge becomes zero, i.e., when the imaginary part of the total impedance vanishes $\mathrm{Im}(f(x,a,b,c)) = 0$. The corresponding condition can be expressed as:

$$x_0(a,b,c)^2 = \frac{(a+c)^2(1-b^2)}{(b+c)^2(1-a^2)}, \quad a \neq 1 \ , \ b \neq 1 \ . \tag{5}$$

The frequency of phase resonance retains both geometric and parametric symmetries of the bridge: $x_0(b,a,c) = 1/x_0(a,b,c)$ and $x_0(1/a,1/b,1/c) = x_0(a,b,c)$. The found formula gives the conditions for the existence of phase resonance $(1-b^2)(1-a^2) > 0$, that is $0 < a < 1$ and $0 < b < 1$or $a > 1$ and $b > 1$. Notably, the existence of phase resonance is independent of the resistance $R_3$ located on diagonal BD. The special case c → ∞ corresponds to configurations previously analysed in [2,3,4,5].

5. ***Amplitude Resonance***. Amplitude resonance is defined as the frequency at which the magnitude of current through the bridge reaches an extremum – maximum for current resonance or minimum for voltage resonance. The condition for amplitude resonance follows from minimizing or maximizing the squared modulus of *f(x, a, b, c)*:

$$F(x,a,b,c) = |f(x,a,b,c)|^2 = \frac{x^2(a+b+c+abc)^2 + \left(a(b+c)x^2 - b(a+c)\right)^2}{x^2(1+ab+ac+bc)^2 + \left((b+c)x^2 - (a+c)\right)^2} \tag{6}$$

This function also preserves both of our symmetries: $F(x,a,b,c) = F(-1/x,b,a,c)$ and $F(x,1/a,1/b,1/c) = 1/F(x,a,b,c)$. Fig. 3 illustrates the amplitude–frequency characteristics for four dual bridge configurations at selected parameter values
(a = 0.6, b = 0.8, c = 0.3). Two of the dual systems exhibit identical resonance frequencies, while the remaining two have reciprocal resonance frequencies.

In each pair, one configuration demonstrates voltage resonance and the other current resonance.

The amplitude resonance frequency satisfies the following equation:

$$w^4(a^2N^2 - M^2 + 2a(b-a)) - 2w^2(b^2 - a^2) - (b^2N^2 - M^2 + 2b(a-b)) = 0 \tag{7}$$

Where the notations are used:

$$N^2 = \frac{(1+ab+ac+bc)^2}{(a+c)(b+c)}\ , \quad w^2 = x^2\frac{b+c}{a+c}\ , \quad M^2 = \frac{(a+b+c+abc)^2}{(a+c)(b+c)}. \qquad (8)$$

From the equation (7), we can determine the conditions for the existence of amplitude resonance of the circuit under consideration:

$$(a^2N^2 - M^2 + 2a(b-a))(b^2N^2 - M^2 + 2b(a-b)) > 0 \qquad (9)$$

### *6. Special Cases and Analytical Observations.*

The derived equations behave consistently under limiting conditions, allowing parameters to take values 0 or ∞. For $c = 0$ and $c = \infty$, the equations admit elegant symmetric solutions:

$$x_0(a,b,\infty)^2 = \frac{\sqrt{1+ab+2b^2} - b^2\sqrt{1+ab+2a^2}}{\sqrt{1+ab+2a^2} - a^2\sqrt{1+ab+2a^2}} \qquad (10)$$

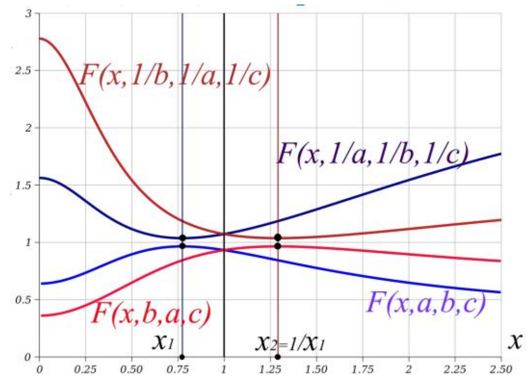


Fig.3 Amplitude-frequency characteristics of four dual circuits. *a*=0.6, *b*=0.8, *c*=0.3

$$x_0(a,b,0)^2 = x_0(1/a,1/b,\infty)^2 = \frac{\sqrt{1+1/(ab)+2/b^2} - \text{[illegible]}}{\sqrt{1+1/(ab)+2/a^2} - 1/a^2\sqrt{1+1/(ab)+2/a^2}}$$

(11)

Transition to reciprocal parameters is physically meaningful, as $c = \infty$ represents a parallel LC connection, while $c = 0$ corresponds to a series LC configuration. The quality factors of these configurations are mutually reciprocal.

For $a = b = 1$, the function $f(x,1,1,c) \equiv 1$ becomes constant, implying frequency-independent current amplitude and zero phase shift. In the neighbourhood of this point, both amplitude and phase resonance frequencies exhibit bifurcation behaviour [5].

When $a = b \neq 1$(symmetrical bridge), the resonance condition simplifies to $x = 1$.

For a balanced bridge $Z_1Z_3 = Z_2Z_4$, our parameters give us the condition $a \cdot b = 1$. We can verify that neither phase nor amplitude resonance occurs for any value of $c$

***7. Conclusion.*** The analysis demonstrates that duality and symmetry principles provide a powerful framework for describing the frequency response of bridge circuits. The established relationships allow the prediction of resonance conditions without the need for separate derivations for each circuit configuration. Furthermore, the observed bifurcation phenomena suggest that small parameter variations can lead to abrupt changes in resonance behaviour, which is of interest in the design of tunable analog filters and oscillatory circuits [6,7].

A comprehensive theoretical model for analysing resonant phenomena in AC bridge circuits has been developed. Analytical expressions for amplitude and phase resonance frequencies have been derived, along with conditions for their existence. The study reveals multiple structural dualities within the bridge topology, forming an Abelian group of transformations that preserve resonance properties. These findings contribute to a deeper understanding of bridge network dynamics and open possibilities for the design of advanced analog filtering systems.

**References**


1. Noel M., Morris. *Electrical Circuit Analysis and Design*. Macmillan, 1993.
2. Блохін Д. О., Демішонкова С. А. Amplitude and phase resonance in a parallel circuit. *Мехатронні системи: інновації та інжиніринг*: матеріали VIII Міжнар. наук.-практ. конф. Київ: КНУТД, 2024. С. 51–52.
3. Блохін Д. О., Демішонкова С. А. Знаходження точногозначення резонансної частоти реального паралельного RLC-контура. *Електромеханічні, інформаційні системи та нанотехнології*: матеріали III Міжнар. наук.-практ. Інтернет-конф. молодих учених та студентів (Київ, 2024). Київ: КНУТД, 2024. С. 50–51.
4. Blokhin D. Duality and resonance in RLC–circuits. Exact formulas for phase, amplitude resonance and bandwidth. *Proc. 2025 IEEE 28th Int. Symp. on Design and Diagnostics of Electronic Circuits and Systems (DDECS)* (Lyon, France, 2025). 2025. P. 165–168.
5. Blokhin D., Demishonkova S. Exact calculation of resonant frequency in a real parallel RLC circuit. *Proc. 2025 Int. Conf. on Electrical, Computer and Energy Technologies (ICECET)* (Paris, France, 2025). 2025. P. 532–537.
6. Stuart A., Lampard D. Bridge Networks Incorporating Active Elements and Application to Network Synthesis. *IEEE Trans. Circuit Theory*. 1963. Vol. 10, no. 3. P. 357–362.
7. Ferguson J. G. Classification of bridge methods of measuring impedances. *Bell Syst. Tech. J.* 1933. Vol. 12, no. 4. P. 452–468.